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SINUSOIDAL VIBRATION TESTING OF NONLINEAR SPACECRAFT STRUCTURES

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SUMMARY

The vibration testing of large spacecraft structures in accordance with procedures similar to those developed for military equipment during the 1940's has numerous shortcomings, particularly if the testing process is considered to be a duplication of an equipment's in-service vibration. The purpose of this technical note is to discuss some of the shortcomings associated with the familiar sinusoidal sweep test.

Waveform distortion, being one of the more obvious problems, is discussed. An analytical model of a simplified structure undergoing vibration testing was studied with the aid of an analog computer. Solutions for a nonlinear model demonstrate distortion of the armature acceleration even though the applied force is sinusoidal. Filtering the control signal to eliminate distortion may unduly penalize the specimen, although this technique is acceptable where the distortion is the "random" type caused by the banging of parts.

The current trend toward larger spacecraft structures will undoubtedly continue, and the problems we now face will be small in comparison to those of the future unless some revisions are made in today's philosophy.

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INTRODUCTION

The sinusoidal sweep vibration test is required by Goddard Space Flight Center (GSFC) for the qualification of spacecraft structures and subassemblies. Although it is generally agreed that random vibration testing more closely simulates the actual flight environment, sinusoidal tests will continue to be specified either as a supplement to the random test or, in some cases, as the sole vibration requirement. The reason for this policy is that the sinusoidal test offers certain advantages over the random test:

1. The sinusoidal test is superior as a diagnostic tool. Since excitation is applied at a single frequency, resonant frequencies and modes can be accurately described. Because random excitation produces the simultaneous response of many modes, the behavior of each mode is obscured. Performed in the development stage, the sinusoidal test invariably points out design deficiencies that can be corrected early in the test program.

2. Sinusoidal vibration can be applied in the frequency ranges not included in typical random tests. Most important is the frequency range of 5 to 20 cps, in which stress levels are likely to be high and interaction of a spacecraft and a vehicle, in its low frequency modes, is likely to occur. Many of the larger spacecraft now being developed have resonances below 20 cps.

3. Sinusoidal testing is relatively inexpensive, and equipment is readily available. For this reason, testing at the subassembly level is often accomplished by using only sinusoidal excitation. Sine wave tests are usually specified in terms of motion (acceleration, velocity, or displacement) at the normal mounting point of the equipment under test.* One of the major problems occurring in sinusoidal vibration testing of structures that are heavy compared with the shaker armature is waveform distortion in the motion of the mounting point.

*One exception is the specification for the NASA Scout and Delta payloads, in which there is an option allowing simulation of the solid rocket motor vibration by controlling the force imparted to the payloads.

Since the "input," or specification level, is normally monitored at the mounting point, waveform distortion here raises questions about the adequacy of the test. Figures 1 and 2 show examples of distorted waveforms that were observed during the vibration testing of the Orbiting Solar Observatory I satellite (1962 ζ1) structural model.*

The purpose of this report is to review some possible sources of waveform distortion, present the results of an analog study of a nonlinear system that exhibited distortion, and discuss the effects of distortion on vibration testing.

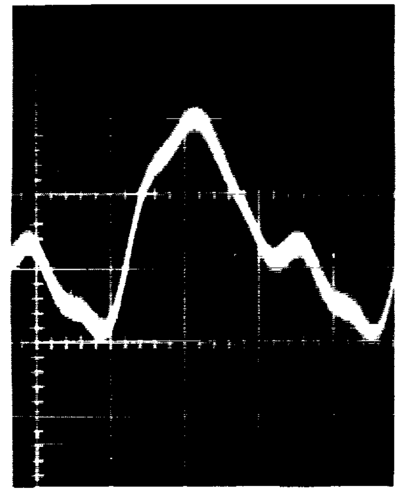

Figure 1—Harmonic distortion of "input."

SOURCES OF DISTORTION

The problem of distortion in sinusoidal vibration testing is not a new one. However, its magnitude has increased in recent years until it can no longer be ignored as it has in most cases in the past. In some recent tests,† the harmonic components have exceeded the fundamental, so that determination of the source of the distortion has became imperative.

Wrisley (Reference 1) suggests that in some cases the equipment can be at fault. If a small amount of harmonic distortion is present in the output of the vibrator's power supply, we can expect a condition in which the frequency of a harmonic is coincident with that of a lightly damped resonance in either the structure being tested or the armature. The resonant structure could then be excited at a level great enough to produce a significant amount of harmonic motion at the "input" transducer location.

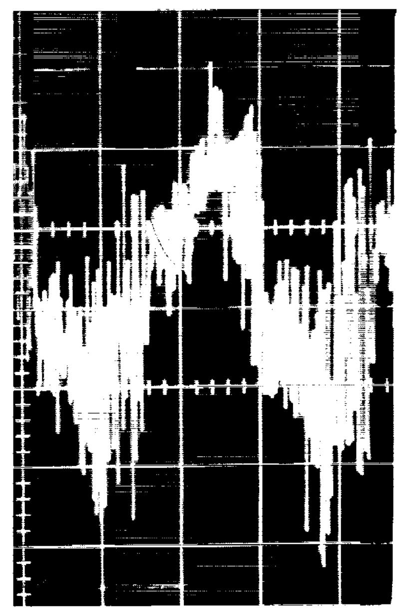

Figure 2—Aperiodic distortion of "input."

Although waveform distortion in the electrical input can certainly be a cause of "input" motion distortion, the type of distortion plaguing the engineer in spacecraft testing is that resulting from structural nonlinearities. The following symptoms support this theory:

1. The frequency of the harmonic usually does not correspond to a resonant frequency of the structure.

*Kirchman, E. J., and Hartenstein, R., "Evaluation of Vibration Test Data from the S-16 Structural Model Tests," NASA Report 321-1 (RH) S-16-09, Goddard Space Flight Center, May 1961.
†For example, Shockey, E. F., "S-51 Dutchman-Separation Mechanism Vibration Tests," NASA/GSFC Memorandum Report 621-7, December 16, 1961.

2. The apparent resonant frequency varies with the amplitude of excitation. This is a well-known characteristic of nonlinear structures.

3. The "random" distortion (as shown in Figure 2) couldn't very well be attributed to electrical wave distortion.

Structures exhibiting nonlinear stiffness properties can be broadly classified as either continuous or discontinuous. An example of a *discontinuous* structure is one in which there is small clearance or looseness between parts. If, during the vibration excitation, parts collide, many modes of the parts will be excited at high accelerations. Figure 2 shows the effect of this phenomenon on the "input" acceleration.

Structures that are *continuously* nonlinear influence the shaker motion by adding harmonics to the waveform. Figure 1 is an example of this. To further understand the effects of nonlinear structures undergoing vibration, a simplified shaker and a single-degree-of-freedom specimen with a cubic hardening spring were studied by means of an analog simulation.

ANALOG SIMULATION

The mathematical model is based on the following assumptions:

1. The vibrator's armature, the test fixture, and the part of the test specimen not resonating act as a rigid mass.

2. The part of the specimen in resonance can be represented as a mass with a nonlinear connecting spring.

3. The force acting on the vibrator's armature coil is sinusoidal regardless of the motion of the armature.

The system is shown in Figure 3. Summing the forces on each mass yields the equations of motion:

$$M\ddot{x} + C\dot{x} + D(\dot{x} - \dot{y}) + Kx + f[x - y] = F \sin\omega t \ , \quad (1)$$

$$m\ddot{y} + D(\dot{y} - \dot{x}) + f[y - x] = 0 \ , \quad (2)$$

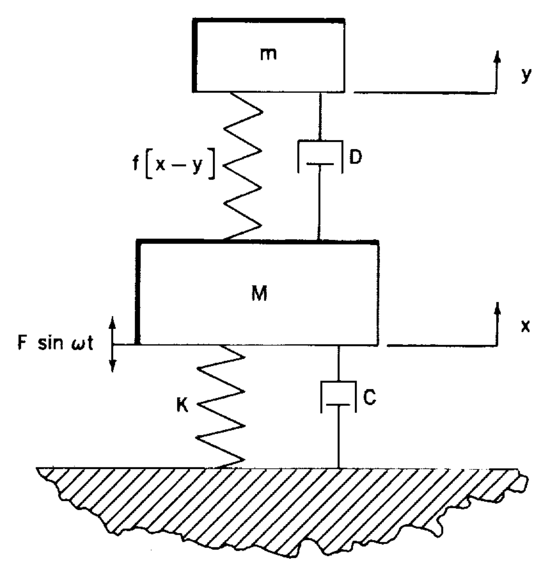

Figure 3—Mathematical model of vibration exciter and nonlinear specimen.

where f [ ] is the nonlinear spring force, a function of the spring extension $(y - x)$ or compression $(x - y)$, and the notation for masses, spring constant, damping coefficients, and coordinates is indicated in Figure 3.

Structures often have a tendency to become stiffer or to "harden" with deflection. Thin panels are known to behave in this manner (Reference 2). The Duffing spring, represented by a linear plus a cubic term in the force deflection expression, has been used to represent the structural stiffness. Thus, $f[x - y]$ in Equation 1 has been taken as

$$\left.\begin{aligned} f[x - y] &= k(x - y) + \beta(x - y)^3 , \\ f[y - x] &= - f[x - y] . \end{aligned}\right\} \quad (3)$$

Equations 1 and 2 can be expressed in dimensionless form by making use of Equation 3 and the following identities:

$$\tau = \omega_0 t \ , \ \omega_0{}^2 = \frac{k}{m} \ , \ \Omega^2 = \frac{K}{M} \ , \ \frac{C}{C_c} = \frac{C}{2\Omega M} \ , \ \frac{D}{D_c} = \frac{D}{2\omega_0 m} \ ;$$

$$x = \left(\frac{g}{\omega_0{}^2}\right)\phi \ , \qquad y = \left(\frac{g}{\omega_0{}^2}\right)\psi \ ,$$

$$\dot{x} = \left(\frac{g}{\omega_0}\right)\frac{d\phi}{d\tau} \ , \qquad \dot{y} = \left(\frac{g}{\omega_0}\right)\frac{d\psi}{d\tau} \ ,$$

$$\ddot{x} = g\,\frac{d^2\phi}{d\tau^2} \ , \qquad \ddot{y} = g\,\frac{d^2\psi}{d\tau^2} \ ;$$

where g is the acceleration of gravity.

By substitution, Equations 1 and 2 become

$$\frac{d^2\phi}{d\tau^2} + 2\,\frac{\Omega}{\omega_0}\,\frac{C}{C_c}\,\frac{d\phi}{d\tau} + 2\,\frac{m}{M}\,\frac{D}{D_c}\,\frac{d}{d\tau}\,(\phi - \psi) + \frac{\Omega^2}{\omega_0{}^2}\,\phi + \frac{m}{M}\,(\phi - \psi) + \frac{m}{M}\,\frac{\beta}{k}\,\frac{g^2}{\omega_0{}^4}\,(\phi - \psi)^3 = \frac{F}{Mg}\sin\frac{\omega}{\omega_0}\,\tau \ , \quad (4)$$

$$\frac{d^2\psi}{d\tau^2} + 2\,\frac{D}{D_c}\,\frac{d}{d\tau}\,(\psi - \phi) + (\psi - \phi) + \frac{\beta g^2}{k\omega_0{}^4}\,(\psi - \phi)^3 = 0 \ . \quad (5)$$

Most of the dimensionless coefficients in Equations 4 and 5 are simple ratios that need no explanation. The significance of the term $\beta g^2/k\omega_0{}^4$, however, isn't immediately obvious. If the substitution for the static deflection δ of a mass resting on a linear spring (of rate k) is made,

$$\frac{\beta g^2}{k\omega_0{}^4} = \frac{\beta}{k}\,\delta^2 = \frac{\beta\delta^3}{k\delta} \ .$$

Thus, $\beta g^2/k\omega_0^4$ is the ratio of the nonlinear force component to the linear force at the deflection δ. Figure 4, which shows the force deflection curves for the linear and the Duffing springs, better illustrates the significance of the parameter. It should be emphasized that the deflection δ is the static deflection for a linear spring (i.e., mg/k) and not the actual deflection for the Duffing spring.

An analog computer was used to obtain some solutions to Equations 4 and 5. The Duffing spring characteristic was obtained with a diode function generator. Seven connected straight line segments approximated the force deflection curve for the spring. Thus the accuracy of the solutions, for very low amplitudes, leaves something to be desired. Where $\phi - \psi$ is not small, several line segments are being utilized and the approximation is adequate.

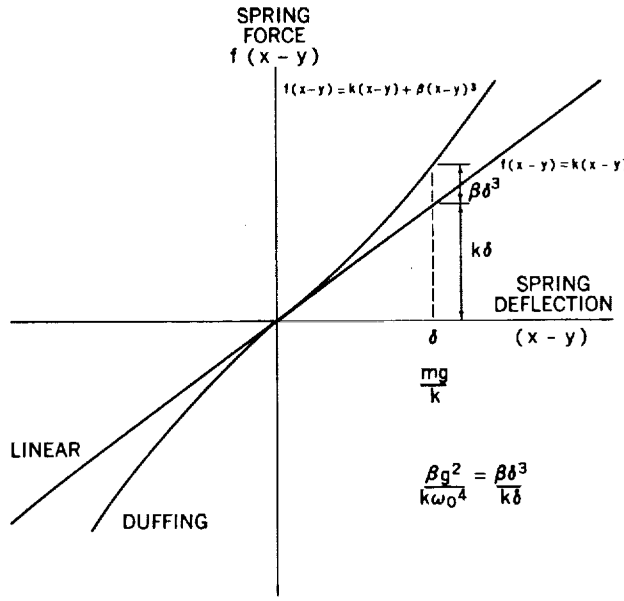

Figure 4—Force deflection curves for the linear and the Duffing springs.

To obtain a solution to Equations 4 and 5, specific numerical values had to be selected for the coefficients. The following values were selected as possibly representing an actual system:

$$\frac{C}{C_c} = 0.05 , \qquad \frac{m}{M} = 0.50 ,$$

$$\frac{D}{D_c} = 0.05 , \qquad \frac{\beta g^2}{k\omega_0^4} = 0.10 ,$$

$$\frac{\Omega}{\omega_0} = 0.20 .$$

With these parameters set into the computer, a sinusoidal input $\frac{F(\omega)}{Mg} \sin \frac{\omega}{\omega_0}\tau$ was applied such that the peak nondimensional acceleration $d^2\phi/d\tau^2$ was approximately constant for the forcing frequency range of ω/ω_0, varying from 0.7 to 1.38. Some resulting waveforms are shown in Figure 5 for the three values of zero to peak acceleration: 0.25, 0.50, and 0.75.

It is also interesting to note how the characteristics of the model vibrator change when the usually assumed linear load is replaced with a nonlinear load. Here, the sinusoidal force amplitude was held constant as frequency was varied. The armature acceleration $d^2\phi/d\tau^2$ was monitored, and the response curves for four forcing amplitudes are presented in Figure 6. The zero to peak values of $d^2\phi/d\tau^2$ were plotted after dividing by F/Mg to normalize the curves. The curve for the near-zero force corresponds to the well-known linear solution in which the resonant specimen influences the

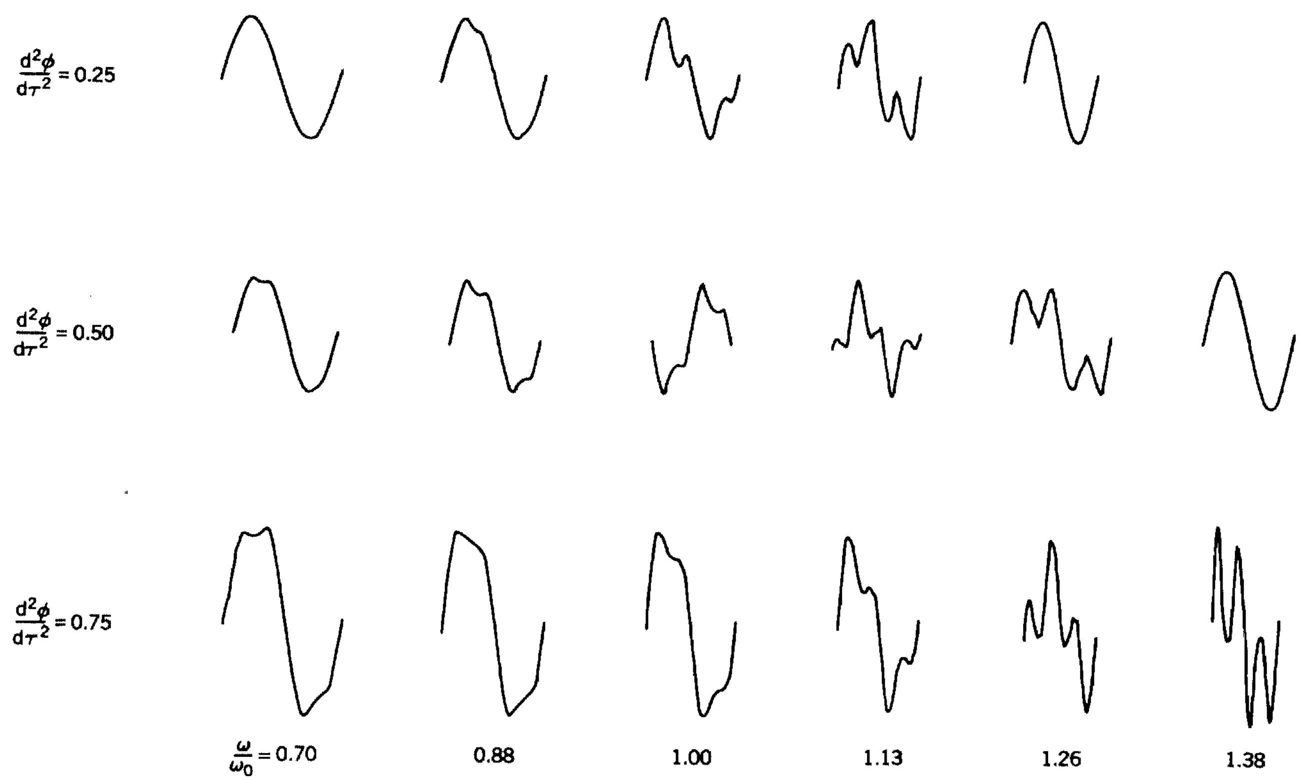

Figure 5—Wave shapes for $d^2\phi/d\tau^2$ (peak) = 0.25, 0.50, 0.75.

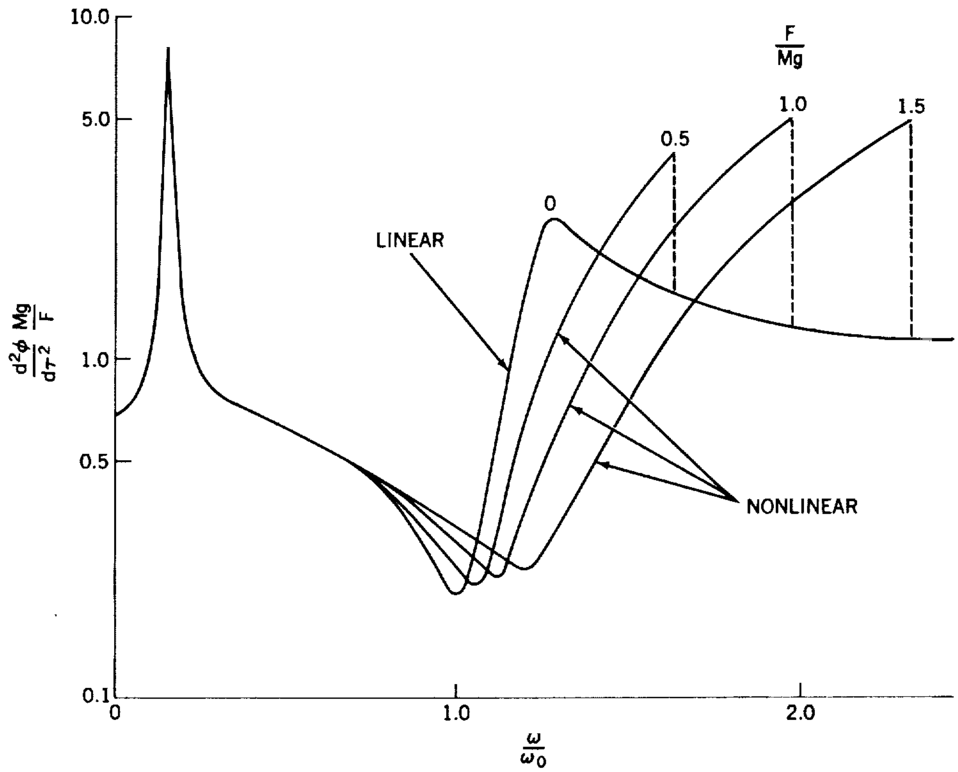

Figure 6—Vibration acceleration for a constant force input $\frac{d^2\phi}{d\tau^2}\frac{Mg}{F}$ vs. $\frac{\omega}{\omega_0}$.

vibrator's characteristics by the insertion of a notch and peak in the response curve. The distortion of the frequency response curve for the nonlinear case is clearly shown in Figure 6. The waveform of $d^2\phi/d\tau^2$ is a distorted sine wave for all of the nonlinear curves. As the driving frequency is increased, a point is reached where the system response abruptly changes. The deflections of the nonlinear spring become small, and the system behaves very nearly as a linear system.

DISCUSSION

The analysis has explained one of the causes of distortion that occurs during a sinusoidal test. Experience with large spacecraft structures has shown that the nonlinear structural phenomenon is the most important one, namely because it appears at the major structural resonances where large excursions and high stress levels are likely to cause structural failure. On smaller packages the effect of nonlinear stiffness of the package isn't too important because the major resonances occur at higher frequencies with lower stress levels and also because the harmonic forces generated within the small structure are not capable of driving the relatively large mass of the shaker armature and fixture at significant amplitudes.

The usefulness of the analysis ends here since, by the nature of nonlinear problems, a solution for a given set of parameters cannot readily be extended to another problem. Also, the nonlinear parameters for a real structure are extremely difficult to define.

The question that naturally follows an explanation of the source of a problem is what to do about it. Suggestions from literature on the subject are summarized below:

1. Wrisley (Reference 1) attributes most distortion to harmonics in the amplifier output that may be multiplied when one of the harmonics corresponds to the armature or some other structural resonant frequency. He points out the errors in trying to measure and control peak acceleration by using an averaging meter for varying percentages and phase of third harmonic distortion. Wrisley concludes that vibration facilities should be equipped to use feedback proportional to the peak acceleration and a true peak reading meter to monitor acceleration.

2. Schafer (Reference 3) points out that vibration amplitudes may be in error by ±90 percent because of distorted acceleration waves. He feels that the insertion of a filter on the output of the control accelerometer, to eliminate all but the driving frequency component of acceleration waveform, would result in a test that comes closer to carrying out the intent of the specification.

Apparently these authors disagree, since Wrisley proposes that the instrumentation be such that the actual peak is sensed and used for control—regardless of the frequency components contributing to the peak, whereas Schafer feels that everything but the fundamental should be disregarded.

When a specification requires a given acceleration in the sinusoidal test schedule, it seems reasonable to assume that this level applies to the fundamental forcing frequency even if the motion cannot be maintained sinusoidal. Schafer's approach, then, is the obvious choice if the test is to be carried out in strict accordance with the specification.

There are cases, especially when testing large structures, where presently available vibration equipment is being driven at maximum force output and still not meeting the specified acceleration levels, even including harmonics. In general, this situation arises at the major resonances of the structure where distortion is most likely to occur. If the recommendation given by Schafer is followed, the deficiency of available force will be even greater. Likewise, the percentage of harmonic content will increase since it is generated by nonlinear phenomena.

Specifications can be satisfied if substantial changes are made in the basic vibrator design. First of all, force output could be increased to the 100,000 pound range. Armature weight, presently kept to a minimum, could be increased substantially, and thereby increase the mechanical impedance of the shaker table and reduce the effect of harmonic forces on "input" motion.

The thought of applying a vibratory force of even 30,000 pounds (today's limit) to a rocket-borne payload structure should raise a question regarding the soundness of the philosophy behind such requirements: What are the requirements dictating that a structure be qualified by the following procedure?

1. Attaching it to another structure that is unspecified and drastically different from the launch vehicle,

2. Exciting it by a force that is generally unknown and that is controlled only by the response of a point where the spacecraft structure and vibrator structure are joined, and

3. Limiting the input force to the maximum force capability of the vibrator.

The answer to this question probably lies in the evolution of vibration testing. The philosophy in the early days was to "drive" a relatively small article with a high impedance device: either a mechanical shaker, or the electrodynamic machines available at the time. Here, the assumptions were simply that the environment could be simulated by duplicating motion because of the high impedance of the equipment's normal mounting structure as well as the shaker.

Today there is no technical justification for motion-controlled testing; it exists only because of its history. The reference to the shaker's acceleration as an "input" demonstrates the need for revision in our thinking. In large test specimens, the shaker table's motion is no more an "input" than is the motion of the end of the structure. In some cases, it's likely to be a node; in others, an antinode. The major resonances observed during the test are usually nonexistent when the spacecraft is mated with its vehicle.

The problem of waveform distortion presents a dilemma that has to be resolved by the originator of the specification rather than the vibration equipment manufacturer. It is he who has failed to recognize the fact that, in testing real structures, nonlinearities exist and deviations from the required test are natural occurrences. This problem, which is one of many associated with vibration simulation, can be resolved only through a revision of present requirements.

CONCLUDING REMARKS

Test concepts for the future should include considerations for monitoring force as well as motion; for the effect of the impedance of the body being tested, the launch vehicle, and the vibrator; for control of the vibrator's impedance, if feasible; for reliance on vehicle/spacecraft analyses for loads and mode shapes that cannot be duplicated but certainly should be considered in the laboratory. The possibility of eliminating shaker-driven vibration testing above 200 cps, for example, and reliance on acoustic testing in the high frequency range should also be considered where applicable. Testing with the structure cantilevered from a solid foundation, for more controlled testing and the acquisition of structural dynamic properties not distorted by shaker and slip table characteristics, might more closely simulate in-flight load distributions.

A few procedures that are recommended by GSFC for the testing of large spacecraft structures might be useful to those presently facing the problem with similar structures:

1. Waveform should be closely monitored so that we can at least know what happened during the test—right or wrong. This, by necessity, requires that data be stored on magnetic tape and spectrum-analyzed at critical frequencies.

2. The signal from the control transducer should not be filtered unless it has been established that the distortion is that resulting from looseness or banging of parts. On a satellite structure recently tested at GSFC, the second harmonic of the table acceleration was 3.8 times that of the forcing frequency.* In this case, filtering and driving the fundamental to the specified level would surely have destroyed the structure.

3. Since motion control can result in unreasonably high loads, fixtures should be equipped with force transducers to monitor bending moment and axial load. These loads should not be allowed to exceed the design limit during the vibration test.

ACKNOWLEDGMENTS

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*See footnote re Shockey report, p. 2.

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